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RABOTNOV'S TWO-LAYER MODEL OF A SHELL AND CRITICAL TIME OF SHELL BUCKLING DURING CREEP

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The main propositions of Rabotnov's two-layer model of a shell are given. It is shown that the Rabotnov's functional can be obtained from the mixed variational principle of creep theory. The notion of critical time is introduced and a procedure for obtaining an explicit formula for it using a variational equation is described.

Key words: *two-layer shell model, functional, creep, critical time.*

Introduction. Rabotnov proposed a two-layer shell model [1] to solve problems of elastoplastic deformations. In [2, 3], the model was extended to the case of deformation of shells under creep conditions. In [3], the class of shells is considered for which the use of the two-layer model cannot introduce large errors to the solution of particular problems. This class of shells is described by the so-called engineering theory of shells which includes the theory of axisymmetric deformation of a circular cylindrical shell, the theory of long cylindrical shells, and the theory of pure bending of a curvilinear tube. The use of the two-layer shell model made it possible to develop an effective numerical method for solving the problems of the stress–strain state of shells under creep conditions [4]. Extension of the two-layer model to the case of shells reinforced by ribs is given in [5–7].

1. Rabotnov's Two-Layer Model of a Shell. The model is a shell consisting of two layers, each of which has thickness h_1; the distance between the mid-surfaces of the layers is $2h$ (see Fig. 1). It is assumed that the layers are connected with each other by a filler which transmits shear forces but does not perceive tension and bending moments. Deformation of the layers occurs in accordance with to the Kirchhoff–Love hypothesis. The surface equidistant from the bearing layers is taken to be the reference surface. The stresses over the thickness of the bearing layers are considered constant.

Let us find the relationship between the parameters of the model h_1 and h and the real-shell thickness $2H$, assuming that the properties of the materials of the model and shell are identical. This relationship is established on the basis of the requirement that the behavior of the model and the real shell coincide for both a momentless stress state and cylindrical bending.

The steady-sate creep relations can be written as $\sigma_{ij} = s(v)\,\partial v/\partial\varepsilon_{ij}$ [σ_{ij} are the stress-tensor components, ε_{ij} are the creep rate tensor components, s is the stress intensity, and v is the intensity of the shear velocity; the function $s(v)$ is determined from data obtained in uniaxial-tension experiments]. Below, we consider a plane stress state. In the case of a momentless stress state, the strain rates ε_{ij} are constant along the thickness in the real shell and are identical for the upper and lower layers in the model. In the real shell, the forces are expressed in terms of the strain rate by the formula

$$T_{11} = \frac{8}{3}\,\frac{s(\varepsilon_0)}{\varepsilon_0}\,H\Big(\varepsilon_{11} + \frac{1}{2}\,\varepsilon_{22}\Big), \tag{1.1}$$

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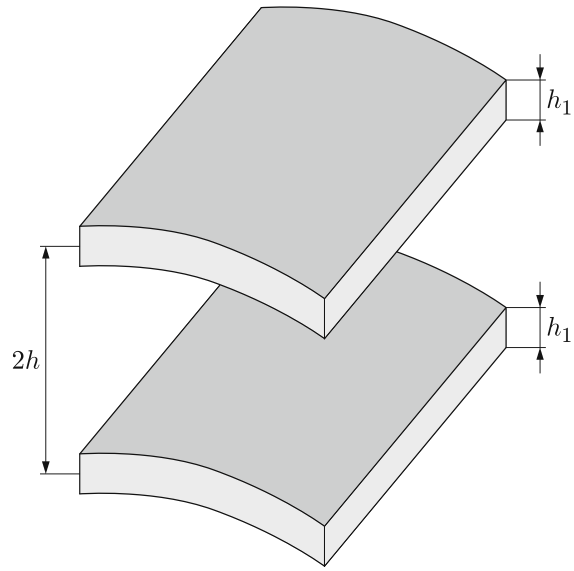

Fig. 1. Two-layer shell model.

and in the model, they are defined by the formula

$$T_{11} = \frac{8}{3}\,\frac{s(\varepsilon_0)}{\varepsilon_0}\,h_1\Big(\varepsilon_{11} + \frac{1}{2}\,\varepsilon_{22}\Big). \tag{1.2}$$

Here $\varepsilon_0 = \sqrt{\varepsilon_{11}^2 + \varepsilon_{22}^2 + \varepsilon_{11}\varepsilon_{22}}$.

From Eqs. (1.1) and (1.2), we obtain

$$h_1 = H. \tag{1.3}$$

In the case of pure bending of the shell $\varepsilon_{ij} = 2\varkappa_{ij}$ ($\varkappa_{ij}$ are the rates of change in the curvatures of the midsurface in the real shell and the reference surface in the model). In this case, the strain rate intensity is $\varepsilon_0 = \varkappa_0 z$ ($\varkappa_0$ is the invariant expressed in terms of the rates of change in the curvatures in the same manner as ε_0 is expressed in terms of ε_{ij}). Hence, the stress intensity is a function of $\varkappa_0 z$ [$\sigma_0 = s(\varkappa_0 z)$].

Thus, for the real shell, we obtain

$$M_{11} = \int\limits_{-H}^{H} \sigma_{11} z\,dz = \frac{8}{3\varkappa_0}\Big(\varkappa_{11} + \frac{1}{2}\,\varkappa_{22}\Big)\int\limits_{0}^{H} s(\varkappa z) z\,dz, \tag{1.4}$$

and for the model, we obtain

$$\varepsilon_{ij}^{\pm} = \pm h\varkappa_{ij}, \qquad \sigma_{ij}^{\pm} = \pm\frac{4}{3}\,\frac{s(\varkappa_0 h)}{\varkappa_0}\Big(\varkappa_{11} + \frac{1}{2}\,\varkappa_{22}\Big).$$

In the two-layer shell, the bending moment is calculated by the formula

$$M_{11} = 2\sigma_{11} h_1 h = \frac{8h_1 h s(\varkappa_0 h)}{3\varkappa_0}\Big(\varkappa_{11} + \frac{1}{2}\,\varkappa_{22}\Big). \tag{1.5}$$

From Eqs. (1.4) and (1.5), it follows that

$$h_1 h s(\varkappa_0 h) = \int\limits_{0}^{H} s(\varkappa_0 z) z\,dz. \tag{1.6}$$

For the exponential creep law $\varepsilon = \varepsilon_* s(\sigma/\sigma_*)^n$, relation (1.6) becomes

$$h_1 h^{1+1/n} = \frac{n}{1+2n}\,H^{2+1/n}.$$

Using (1.3), we obtain

$$h = \left(\frac{n}{1+2n}\right)^{n/(n+1)} H. \tag{1.7}$$

Relation (1.7) contains the creep exponent n. As n varies from 1 to ∞, the value of h varies in the range of $1 \le h \le H/\sqrt{3}$. The value $n = \infty$ corresponds to the case of ideal plasticity.

Using the Kirchhoff–Love hypothesis, the strain rates in the bearing layers of the shell are written as

$$\varepsilon_{11}^{+} = \varepsilon_{11} + \varkappa_{11} h, \qquad \varepsilon_{11}^{-} = \varepsilon_{11} - \varkappa_{11} h, \qquad \varepsilon_{22}^{+} = \varepsilon_{22}^{-} = \varepsilon_{22}, \tag{1.8}$$

where ε_{11} and ε_{22} are the strain rates of the mid-surface; $\varkappa_{11}$ is the rate of change in the curvature in direction 1. In (1.8) and below, the superscript plus corresponds to the upper bearing layer and the minus sign to the lower layer. The stresses in the bearing layers are calculated by the formulas

$$\sigma_{11}^{+} = \frac{4}{3}\,\frac{s^{+}}{v^{+}}\Big(\varepsilon_{11} + \varkappa_{11} h + \frac{1}{2}\,\varepsilon_{22}\Big), \qquad \sigma_{11}^{-} = \frac{4}{3}\,\frac{s^{-}}{v^{-}}\Big(\varepsilon_{11} - \varkappa_{11} h + \frac{1}{2}\,\varepsilon_{22}\Big) \qquad (1 \rightleftarrows 2). \tag{1.9}$$

If $T_{11} = 0$, then $\sigma_{11}^{+} = -\sigma_{11}^{-}$. From (1.9), it follows that the condition $T_{11} = 0$ is satisfied if

$$\varepsilon_{11} + \frac{\varepsilon_{22}}{2} = 0, \qquad \frac{s^{+}}{v^{+}} = \frac{s^{-}}{v^{-}}, \tag{1.10}$$

whence, in turn, it follows that $v^{+} = v^{-}$ (the materials of the layers are considered identical).

If the first condition in (1.10) is satisfied, the stresses in the layers are calculated by the formulas

$$\sigma_{11}^{\pm} = \pm\frac{4}{3}\,\frac{s^{\pm}}{v^{\pm}}\,\varkappa_{11} h, \qquad \sigma_{22}^{\pm} = \frac{s^{\pm}}{v^{\pm}}\Big(\varepsilon_{22} \pm \frac{2}{3}\,\varkappa_{11} h\Big). \tag{1.11}$$

Substituting (1.11) into the expression for the stress intensity, we obtain

$$(v^{\pm})^2 = \varepsilon_{22}^2 + 4\varkappa_{11}^2 h^2/3. \tag{1.12}$$

From (1.10) and (1.12), it follows that $v^{+} = v^{-}$ and $s^{+} = s^{-}$. The force T_{22} and the bending moment M_{11} are calculated by the formulas

$$T_{22} = h_1(\sigma_{22}^{+} + \sigma_{22}^{-}) = \frac{2h_1 s(v)}{v}\,\varepsilon_{22}, \qquad M_{11} = h_1(\sigma_{22}^{+} - \sigma_{22}^{-}) = \frac{8h_1 s(v)}{3v}\,\varkappa_{11} h^2.$$

In the dimensionless variables

$$\omega = \frac{\sigma_* v}{\varepsilon_* s}, \qquad u = \frac{\varepsilon_{22}}{\varepsilon_*}, \qquad m = \frac{\sqrt{3}}{4}\,\frac{M_{11}}{h h_1 \sigma_*}, \tag{1.13}$$

relation (1.12) is written as

$$(v^{\pm}(\omega))^2/\varepsilon_*^2 = u^2 + m^2\omega^2. \tag{1.14}$$

The form of the function $v(\omega)$ depends on the creep law. For the specified creep law, Eq. (1.14) can be resolved for ω. Thus, for the exponential creep law $\varepsilon = \varepsilon_*(\sigma/\sigma_*)^n$, this relation has the form

$$\omega^{2n/(n-1)} = u^2 + m^2\omega^2.$$

The force T_{22} and the rate of change in the curvature $\varkappa_{11}$ are expressed in terms of the dimensionless variables (1.13) as follows:

$$T_{22} = 2h_1\sigma_* u/\omega, \qquad \varkappa_{11} h = \sqrt{3}\,\varepsilon_* m\omega/2.$$

The equations of axisymmetric deformation of a circular cylindrical shell in the dimensionless variables are written as

$$u'' + 2m\omega = 0, \qquad m'' - 2u/\omega + 2p = 0, \tag{1.15}$$

where

$$p = -\frac{q_n R}{2h_1\sigma_*}, \qquad b^2 = \frac{4}{\sqrt{3}}\,Rh,$$

q_n is the uniformly distributed external pressure and R is the shell radius; primes denote derivatives with respect to the dimensionless coordinate $\xi = x/b$.

For independent variation of the quantities u and m, Eqs. (1.15) are the Euler equations for the functional

$$N = \int_0^l [u'm' + \psi(\omega) - 2m^2\omega - 2pu]\, d\xi,$$

where

$$\psi(\omega) = \frac{1}{\varepsilon_*^2} \int \frac{d(v^2)}{\omega}$$

and l is the dimensionless length of the shell.

Equations (1.14) and (1.15) are extended to the case of nonzero longitudinal compressing force ($T_{11} \neq 0$):

$$m'' - u(1/\omega^+ + 1/\omega^+) - 2\tau/\sqrt{3} + 2p = 0, \qquad u'' + (m+\tau)\omega^+ - (m+\tau)\omega^- = 0,$$

$$(v^\pm/v^*)^2 = u^2 + (m \pm \tau)^2(\omega^\pm)^2. \tag{1.16}$$

Here

$$\tau = \sqrt{3}\, T_{11}/(4h_1\sigma_*).$$

In this case, the functional N is written as

$$N = \int_0^l \Big[u'm' + \frac{1}{2}\,\psi(\omega^+) + \frac{1}{2}\,\psi(\omega^-) - (m+\tau)^2\omega^+ - (m-\tau)^2\omega^- + 2\Big(\frac{\tau}{\sqrt{3}} - p\Big)u\Big]\, d\xi. \tag{1.17}$$

2. Mixed Variational Principle of Creep Theory and Rabotnov's Functional. We assume that, under steady-state creep conditions, the stresses and creep strain rates are linked by the relations

$$\varepsilon_{ij} = \frac{\partial \Lambda}{\partial \sigma_{ij}}, \qquad \sigma_{ij} - \sigma\, \delta_{ij} = \frac{\partial L}{\partial \varepsilon_{ij}}, \tag{2.1}$$

where Λ is the stress function, L is the strain rate function $\varepsilon_{ij} = (v_{i,j} + v_{j,i})/2$, and v_i are the velocity components; $i, j = 1, 2, 3$.

Let a body occupying a volume V bounded by a surface S be in steady-state creep under the action of surface forces specified on the part of the surface S_σ; on the part of the surface S_v, displacement velocities are specified, and on the part of the surface $S_{\sigma v}$, some components of the velocity vector and stress tensor are specified. In this case, the stresses and displacement velocities should satisfy the equations

$$\sigma_{ij,j} = 0 \quad \text{in } V, \qquad i, j = 1, 2, 3; \tag{2.2}$$

$$\sigma_{ij} n_j = \bar{T}_i \quad \text{on } S_\sigma, \qquad v_i = \bar{v}_i \quad \text{on } S_v, \qquad v_\gamma = \bar{v}_\gamma, \quad \sigma_{\nu j} n_j = \bar{T}_\nu \quad \text{on } S_{\sigma v}$$
$$(i, j, \gamma, \nu = 1, 2, 3; \quad \gamma \neq \nu). \tag{2.3}$$

In (2.3), the prescribed quantities are denoted by a bar. System (2.1)–(2.3) is a closed system of equations for determining the velocity field and stresses in the region V bounded by the surface S.

We consider the functional

$$N = \int_v (\Lambda - \sigma_{ij}\varepsilon_{ij})\, dV + \int_{S_\sigma} \bar{T}_i v_i\, dS + \int_{S_v} (v_i - \bar{v}_i) T_i\, dS + \int_{S_{\sigma v}} [\bar{T}_\nu v_\nu + (v_\gamma - \bar{v}_\gamma) T_\gamma]\, dS. \tag{2.4}$$

Let us calculate the variation of functional (2.4). Varying the stresses and displacement velocities independently and using relations (2.1), we obtain

$$\delta N = \int_v \Big[\Big(\frac{\partial \Lambda}{\partial \sigma_{ij}} - \varepsilon_{ij}\Big)\delta\sigma_{ij} + \sigma_{ij,j}\, \delta v_i\Big]\, dV + \int_{S_\sigma} (\bar{T}_i - T_i)\, \delta v_i\, dS$$
$$+ \int_{S_v} (\bar{v}_i - v_i)\, \delta T_i\, dS + \int_{S_{\sigma v}} [(\bar{T}_\nu - T_\nu)\, \delta v_\nu + (v_\gamma - \bar{v}_\gamma)\, \delta T_\gamma]\, dS. \tag{2.5}$$

From (2.5), it follows that in the case of the true distribution of stresses and strain rates, $\delta N = 0$. For independent variation of stresses and displacement velocities, consequences of the equation $\delta N = 0$ are the relationships between the stresses and displacement velocities (2.1), the equilibrium equations (2.2), and boundary conditions (2.3). Varying only stresses in functional (2.4) providing that the stress variations satisfy homogeneous boundary conditions, we obtain the principle of the minimum dissipation of the additional power. In view of relations (2.1), the stress function Λ can be treated as a function of the strain rate ε_{ij}. In the case of steady-state creep, the following equality holds

$$\int\limits_v \sigma_{ij}\varepsilon_{ij}\,dV = \int\limits_v (\Lambda + L)\,dV.$$

Substituting this equality into functional (2.4) and varying only the displacement velocities, we obtain the maximum principle for the total power.

We use functional (2.4) to derive the equations of axisymmetric deformation for a circular cylindrical shell. For the two-layer shell model, the relationship between strain rates, forces, and moments can be represented as

$$2\varepsilon_{11} = 2\,\frac{d\dot{U}}{dx} = \frac{1}{2h_1h}\Big[\frac{v^+}{s^+}\Big((M_1 + hT_1) - \frac{1}{2}\,(M_2 + hT_2)\Big) - \frac{v^-}{s^-}\Big((M_1 - hT_1) - \frac{1}{2}\,(M_2 - hT_2)\Big)\Big],$$

$$2\varepsilon_{22} = -2\,\frac{\dot{W}}{a} = \frac{1}{2h_1h}\Big[\frac{v^+}{s^+}\Big((M_2 + hT_2) - \frac{1}{2}\,(M_1 + hT_1)\Big) - \frac{v^-}{s^-}\Big((M_2 - hT_2) - \frac{1}{2}\,(M_1 - hT_1)\Big)\Big], \tag{2.6}$$

$$2\varkappa_{11}h = 2h\,\frac{d^2\dot{W}}{dx^2} = \frac{1}{2h_1h}\Big[\frac{v^+}{s^+}\Big((M_1 + hT_1) - \frac{1}{2}\,(M_2 + hT_2)\Big) + \frac{v^-}{s^-}\Big((M_1 - hT_1) - \frac{1}{2}\,(M_2 - hT_2)\Big)\Big],$$

where

$$(s^\pm)^2 = [(M_1 \pm hT_1)^2 + (M_2 \pm hT_2)^2 - (M_1 \pm hT_1)(M_2 \pm hT_2)]/(4h_1^2h^2).$$

The moment M_2 is defined as a function of M_1, T_1, and T_2 from the condition $\varkappa_{22} = 0$:

$$\frac{v^+}{s^+}\Big((M_2 + hT_2) - \frac{1}{2}\,(M_1 + hT_1)\Big) + \frac{v^-}{s^-}\Big((M_2 - hT_2) - \frac{1}{2}\,(M_1 - hT_1)\Big) = 0. \tag{2.7}$$

In the two-layer shell model, the function Λ is defined as follows:

$$\Lambda = h_1\Big(\int v^+\,dS^+ + \int v^-\,dS^-\Big). \tag{2.8}$$

Relations (2.1) in this model are represented as

$$\varepsilon_{11} = \frac{\partial\Lambda}{\partial T_1}, \qquad \varepsilon_{22} = \frac{\partial\Lambda}{\partial T_2}, \qquad \varkappa_{11} = \frac{\partial\Lambda}{\partial M_1}. \tag{2.9}$$

In view of (2.6)–(2.9), functional (2.4) is written as

$$N = \int\limits_0^l (\Lambda - T_1\varepsilon_{11} - T_2\varepsilon_{22} - M_1\varkappa_{11} + q\dot{W})\,dx + \Phi. \tag{2.10}$$

The expression for the term Φ depends on the form of the boundary conditions at the edges of the shell. Thus, if the edges of the shell are hinged:

$$\dot{W}(0) = M_1(0) = \dot{W}(l) = M_1(l) = 0,$$

then,

$$\Phi = \Big(\bar{T}_1\dot{U} - \frac{dM_1}{dx}\,\dot{W}\Big)\Big|_{x=0}^{x=l}. \tag{2.11}$$

If the edges of the shell are rigidly fixed

$$\dot{W}(0) = \frac{d\dot{W}(0)}{dx} = \dot{W}(l) = \frac{d\dot{W}(l)}{dx} = 0,$$

then,

$$\Phi = \Big(\bar{T}_1\dot{U} - \frac{dM_1}{dx}\,\dot{W} + M_1\,\frac{d\dot{W}}{dx}\Big)\Big|_{x=0}^{x=l}. \tag{2.12}$$

In relations (2.11) and (2.12), $\bar{T}_1$ is the prescribed longitudinal force.

Varying the forces, moments, and displacement velocities in the functional (2.10) independently, we obtain

$$\delta N = \int_0^l \Big[\Big(\frac{\partial\Lambda}{\partial T_1} - \varepsilon_{11}\Big)\,\delta T_1 + \Big(\frac{\partial\Lambda}{\partial T_2} - \varepsilon_{22}\Big)\,\delta T_2 + \Big(\frac{\partial\Lambda}{\partial M_1} - \varkappa_{11}\Big)\,\delta M_1 + \frac{dT_1}{dx}\,\delta\dot{U}$$

$$-\Big(\frac{d^2M_1}{dx^2} - \frac{T_2}{a} - q\Big)\,\delta\dot{W}\Big]\,dx - \Big(T_1\,\delta\dot{U} + M_1\,\delta\,\frac{d\dot{W}}{dx} - \frac{dM_1}{dx}\,\delta\dot{W}\Big)\Big|_{x=0}^{x=l} + \delta\Phi,$$

where

$$\delta\Phi = \Big(\bar{T}_1\,\delta\dot{U} - \frac{dM_1}{dx}\,\delta\dot{W} - \dot{W}\,\delta\,\frac{dM_1}{dx}\Big)\Big|_{x=0}^{x=l}$$

in case (2.11) or

$$\delta\Phi = \Big(\bar{T}_1\,\delta\dot{U} - \frac{dM_1}{dx}\,\delta\dot{W} + M_1\,\delta\,\frac{d\dot{W}}{dx} + \frac{d\dot{W}}{dx}\,\delta M_1 - \dot{W}\,\delta\,\frac{dM_1}{dx}\Big)\Big|_{x=0}^{x=l}$$

in the case (2.12). The equation $\delta N = 0$ with arbitrary variations of the displacement velocities, forces, and moments leads to the equilibrium equations for a symmetrically loaded cylindrical shell, boundary conditions at its ends, and relationships between the strain rate of the mid-surface of the shell, forces, and moments.

Expressing all quantities included in functional (2.10) in terms of M_1, T_1, and $\dot{W}$, we obtain

$$T_2 = -h_1\,\frac{\dot{W}}{a}\Big(\frac{s^+}{v^+} + \frac{s^-}{v^-}\Big) + \frac{1}{2}\,T_1, \qquad 2\dot{\varkappa}_1 h = \frac{1}{2h_1h}\Big(\frac{3}{4}\,\frac{v^+}{s^+}\,(M_1 + hT_1) + \frac{3}{4}\,\frac{v^-}{s^-}\,(M_1 - hT_1)\Big),$$

$$(s^\pm)^2 = \Big(\frac{\sqrt{3}}{4}\,\frac{M_1}{h_1h} \pm \frac{\sqrt{3}}{4}\,\frac{T_1}{h_1}\Big)^2 + \frac{\dot{W}}{a}\Big(\frac{s^\pm}{v^\pm}\Big)^2, \tag{2.13}$$

$$\Lambda = h_1\Big(s^+v^+ + s^-v^- - \int s^+\,dv^+ - \int s^-\,dv^-\Big).$$

Transforming to dimensionless variables in (2.13), we have

$$T_2 = h_1s_n\Big[u\Big(\frac{1}{\omega^+} + \frac{1}{\omega^-}\Big) + \frac{2}{\sqrt{3}}\,\tau\Big], \qquad \dot{\varkappa}_1 = \frac{\sqrt{3}}{4h}\,v_n[(m+\tau)\omega^+ + (m-\tau)\omega^-],$$

$$\Big(\frac{v^\pm}{v_n}\Big)^2 = (m \pm \tau)^2(\omega^\pm)^2 + u^2, \tag{2.14}$$

$$\Lambda = h_1s_nv_n\Big[\Big(\frac{v^+}{v_n}\Big)^2\frac{1}{\omega^+} + \Big(\frac{v^-}{v_n}\Big)^2\frac{1}{\omega^-} - \frac{1}{2}\,\Psi(\omega^+) - \frac{1}{2}\,\Psi(\omega^-)\Big],$$

where

$$\Psi(\omega^\pm) = \int \frac{1}{\omega^\pm}\,d\Big(\frac{v^\pm}{v_n}\Big)^2.$$

Substitution of (2.14) into functional (2.10) yields

$$\bar{N} = \frac{N}{bh_1 v_n s_n} = \int\limits_0^l \Big[mu'' - \frac{1}{2}\,\Psi(\omega^+) - \frac{1}{2}\,\Psi(\omega^-) + (m+\tau)^2\omega^+ + (m-\tau)^2\omega^- - 2\Big(\frac{\tau}{\sqrt{3}} - p\Big)u\Big]\,d\xi + \bar{\Phi}, \qquad (2.15)$$

where

$$\xi = \frac{x}{b}, \qquad b = \Big(\frac{16}{3}\Big)^{1/4}\sqrt{ah}, \qquad p = -\frac{qa}{2h_1 s_n}.$$

A consequences of the variational equation $\delta\bar{N} = 0$ are the equilibrium equation

$$\frac{d^2 M_1}{dx^2} - \frac{T_2}{a} - q = 0$$

and the relation

$$\varkappa_{11} - \frac{\partial\Lambda}{\partial M_1} = 0,$$

which in dimensionless variables have the form

$$m'' - u\Big(\frac{1}{\omega^+} + \frac{1}{\omega^-}\Big) - \frac{2}{\sqrt{3}}\,\tau + 2p = 0, \qquad u'' - (m+\tau)\omega^+ + (m+\tau)\omega^- = 0. \qquad (2.16)$$

The quantities $\omega^\pm$ are implicitly expressed in terms of the deflection velocity u and the moment m from the equations

$$(\omega^\pm)^{2n/(n-1)} = u^2 + (m \pm \tau)^2(\omega^\pm)^2. \qquad (2.17)$$

Functional (2.15) is identical to functional (1.17), and Eqs. (2.16) and (2.17) are identical to Eqs. (1.16).

3. Calculation of the Critical Buckling Time of Cylindrical Shells in Creep. Use of functional (2.15) in problems of shell buckling under creep conditions makes it possible to obtain an explicit expression of the critical time. The time t^* is called the critical time if the maximum deflection of the shell tends to infinity as $t \to t^*$. Therefore, in determining the quantities $\omega^\pm$ from Eq. (2.17), we assume that the determining term is the one containing the deflection velocity:

$$\omega^\pm = u^{(n-1)/n}(1 + \varepsilon^\pm). \qquad (3.1)$$

Here

$$\varepsilon^\pm = \frac{n-1}{2n}\,(m \pm \tau)^2 u^{-2/n}$$

in the case of an exponential creep law.

We use representation (3.1) in functional (2.15) to find the coefficients $\dot{\alpha}(t)$ and $\beta(t)$ which determine the time dependence of trial functions for the displacement velocity u and the moment m. In the calculation of the functional, we will retain only the quantities $\varepsilon^\pm$ to the first power. The differential equations for $\dot{\alpha}(t)$ and $\beta(t)$ which follow from the equations $\partial\bar{N}/\partial\dot{\alpha}(t) = 0$ and $\partial\bar{N}/\beta(t) = 0$, can be represented as

$$\alpha(t) = f(\dot{\alpha}(t), \beta(t)), \qquad \varphi(\dot{\alpha}(t), \beta(t)) = 0$$
$$[f(\dot{\alpha}(t), \beta(t)) \to \infty \quad \text{at} \quad \dot{\alpha}(t) \to \infty]. \qquad (3.2)$$

The form of the functions f and φ depends on the employed shell model [4–7].

The general solution of Eqs. (3.2) is written as

$$t = \int\limits_{q_0}^{q} \frac{df(q, q(\beta))}{q}.$$

The dependence $q(\beta)$ is determined from the equation $\varphi(q, \beta) = 0$, and the constant q_0 from the condition $\alpha(0) = \alpha_0$. The critical time is calculated by the formula

$$t^* = \int_{q_0}^{\infty} \frac{df(q, q(\beta))}{q}.$$

Conclusions. The problems of shell deformation under creep conditions at large strains, in particular, the problem of determining the critical time are complex geometrically and physically nonlinear problems. It seems impossible to obtain a solution of these problems in closed form based on the complete equations. It should also be taken into account that the material constants in the constitutive equations of the creep law are experimentally determined with a large error (about 20–30%); therefore, using simplifying hypotheses and approximate models to solve shell creep problems is quite justified. Rabotnov's two-layer shell model and its generalizations allow one to effectively solve problems of the stress–strain state of shells under creep conditions.

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